PROBLEM OF DETERMINING INTERNAL BOUNDARY CONDITIONS IN MEASURING THE HEAT OF GAS-TURBINE BLADES UNDERGOING COOLING

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This article examines features of the numerical solution of a two-dimensional nonlinear inverse problem of heat conduction in a two-layer region with a movable external boundary.

One of the problems encountered in developing convective heat transfer systems for the blades of gas turbines is experimental determination of the distribution of the heat-transfer coefficient over the surface of the internal cavity. Such a determination makes it possible to then evaluate its thermal stress state under service conditions.

The method in [1] is used to accomplish this goal. Here, the blade is thermostatted in a melt of a high-purity metal and then, at its solidification temperature, blown with air for a fixed period of time. The parameters of the crust which forms during crystallization on the outside surface of the blade are used to calculate the amount of heat which has passed through its wall.

Experiments with full-size blades undergoing cooling have shown that the crust is generally variable in thickness. This is due to differences in the heat-transfer coefficients and temperatures in the channels of the internal cavity, variability of the thickness of the wall and the curvature of the profile, the presence of cooling intensifiers and bridges connecting opposite sides of the blade, and local cooling irregularities such as are seen in jet cooling of the inside surface.

Figure 1 shows the middle section of a typical channel-type blade undergoing cooling and the crust formed on its outside surface during the blowing of air through the channels of the internal cavity.

The derivation of quantitative relations on heat transfer in the internal cavity requires the development of a method of interpreting experimental results which will consider not only design features, but also conventional representations of thermophysical relations and features of calculation of the thermal state.

When numerical methods are used in the thermal state calculations to solve the heat-conduction equation, on the basis of a priori considerations the entire internal cavity is broken down into characteristic sections in which the distributions of the heat-transfer coefficients and air temperature are assumed to be uniform. The mean values of the coefficients for these sections α are determined from criterional relations obtained by generalizing the results of thermal and hydraulic experiments under geometrically similar systems. Thus, when the above-described approach is used, mean values of the heat-transfer coefficient for the sections must be determined.

The above method, together with a preliminary hydraulic experiment conducted to determine the distribution of coolant flow over the channels of the internal cavity, makes it possible to obtain the distribution of the heat-transfer coefficient for a specific design. This makes it unnecessary to adapt to this design the results of a thermal experiment for geometrically similar systems modeling individual sections of the internal cavity.

Thus, we need to determine values, constant over time and on the boundaries ∂D_1, ∂D_2, ..., ∂D_N, of the heat-transfer coefficients α_1, α_2, ..., α_N from the known air temperatures T_{B1}, T_{B2}, ..., T_{BN} in corresponding channels of the internal cavity and from the known position ∂D_Σ^* of the movable boundary ∂D_Σ enclosing the region of the blade $\bar{D}$ and crust Σ formed by the moment τ_m of completion of blowing of the cavity. The temperature at each point of the cavity at the moment $\tau = 0$ is equal to the solidification temperature T_{so} (Fig. 1).

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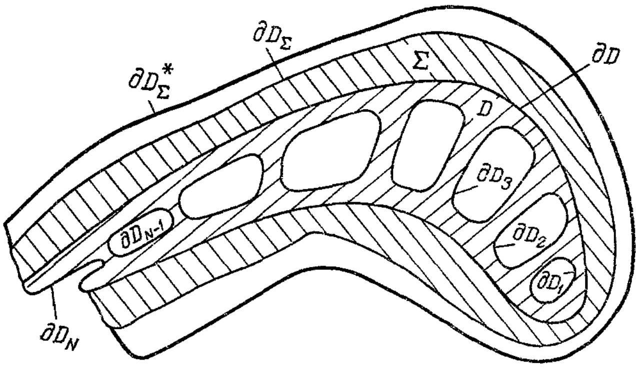

Fig. 1. Middle section of the blade and crust – model of the region of solution of the inverse heat-conduction problem.

The solution of the initial three-dimensional inverse heat-conduction problem (IHP) is made easier by the fact that the blade geometry and, as shown by experiments, the crust on the blade changes little over the longitudinal axis. Heat flow along the axis is negligible compared to heat flow in the transverse direction. This allows us to reduce the problem to a two-dimensional problem in the plane of sections perpendicular to the blade axis.

Assuming that $\alpha_1, \alpha_2, \ldots, \alpha_N$ are constant for the entire time of blowing, we can suggest two basically different methods of determining the heat-transfer coefficient.

The first consists of iterative determination of values of $\alpha_1, \alpha_2, \ldots, \alpha_N$ such that the crust geometry obtained experimentally is attained by the moment of termination of the blowing τ_m. This requires repeated solution of the direct Stefan problem [2], which in regard to blades for modern cooling systems leads to the necessity of developing programs for numerical solution of the Stefan problem in at least the two-dimensional formulation.

Along with the large amount of time required to write these programs, the solution of the inverse problem requires a large amount of computer memory and computing time. This is due to the fact that the thermal conductivity of the high-purity metals used to indicate heat flow through the blade wall is considerably greater than the conductivity of the heat-resistant blade alloys. This in turn makes it necessary to use a finer calculating grid for the crust region Σ, where the configuration of the external boundary changes over time. The large amount of time needed to analyze the results of experiments under these conditions means that a longer time is necessary to obtain the values of $\alpha_1, \alpha_2, \ldots, \alpha_N$.

In our opinion, the second method is preferable. It is based on familiar mathematical techniques [3] and the associated program usually used for two-dimensional calculation of the thermal state of blades undergoing cooling. The method shortens the time required to analyze test data and does not require solution of the two-dimensional Stefan problem in a two-layer with layers having different thermophysical properties. The method also makes calculation of the thermal state under service conditions more accurate, since the mathematical model of the blade section which is used is the same employed to analyze test results.

The method consists of dividing the initial IHP in the region $\bar{D}_\Sigma = \bar{D} + \Sigma + \partial D_\Sigma$ into two IHP's which are solved in succession (Fig. 1).

The first problem is the Cauchy problem – determination of continuous and differentiable functions $f_1(\omega, \tau)$ and $f_2(\omega, \tau)$ on the boundary ∂D from the equation

$$\frac{\partial T(P, \tau)}{\partial \tau} = a_\Sigma\left(\frac{\partial^2 T(P, \tau)}{\partial x^2} + \frac{\partial^2 T(P, \tau)}{\partial y^2}\right), \quad P \in \Sigma, \ \tau \in (0, \tau_m] \tag{1}$$

and the conditions

$$\partial D_\Sigma = \partial D, \ \tau = 0, \tag{2}$$

$$T(\omega, \tau) = f_1(\omega, \tau), \ \omega \in \partial D, \ \tau \in (0, \tau_m], \tag{3}$$

$$\lambda_\Sigma \frac{\partial T(\omega, \tau)}{\partial n_\omega} = f_2(\omega, \tau), \ \omega \in \partial D, \ \tau \in (0, \tau_m], \tag{4}$$

$$\lambda_\Sigma \frac{\partial T(\omega, \tau)}{\partial n_\omega} = q(\omega, \tau), \ \omega \in \partial D_\Sigma, \ \tau \in (0, \tau_m], \tag{5}$$

$$T(\omega, \tau) = T_{so}, \ \omega \in \partial D_\Sigma, \ \tau \in (0, \tau_m], \tag{6}$$

$$\partial D_\Sigma = \partial D_\Sigma^*, \ \tau = \tau_m, \tag{7}$$

where the form of the boundary ∂D_Σ^* is known; λ_Σ and a_Σ are constants; $q(\omega, \tau)$ is a differentiable function, continuous with respect to the corodinates and time, which is determined by the Stefan condition:

$$q(\omega, \tau) = \rho Q \mathbf{V}(\tau) \mathbf{n}_\omega(\tau), \ \omega \in \partial D_\Sigma. \tag{8}$$

The second IHP is determination of the values of $\alpha_1, \alpha_2, \ldots, \alpha_N$ on the boundaries $\partial D_1, \partial D_2, \ldots, \partial D_N$ of the region $\overline{D}$ from the equation

$$\frac{\partial T(P, \tau)}{\partial \tau} = a_D \left(\frac{\partial^2 T(P, \tau)}{\partial x^2} + \frac{\partial^2 T(P, \tau)}{\partial y^2} \right), \ P \in D, \ \tau \in (0, \tau_m] \tag{9}$$

and the conditions

$$T(P, 0) = T_{so}, \ P \in \overline{D}, \tag{10}$$

$$\lambda_D \frac{\partial T(\omega, \tau)}{\partial n_\omega} = \alpha_i (T(\omega, \tau) - T_{si}), \ \omega \in \partial D_i, \ \tau \in (0, \tau_m], \tag{11}$$

$$T(\omega, \tau) = f_1(\omega, \tau), \ \omega \in \partial D, \ \tau \in (0, \tau_m], \tag{12}$$

$$\lambda_D \frac{\partial T(\omega, \tau)}{\partial n_\omega} = f_2(\omega, \tau), \ \omega \in \partial D, \ \tau \in (0, \tau_m], \tag{13}$$

where α_i and T_{si} ($i = 1, 2, \ldots, N$), λ_D, and a_D are constants.

Below we explore the possibility of numerical solution of just the first problem, and we propose a method of obtaining such a solution. The second problem can be solved by well-known methods [4, 5].

The problem being examined is incorrect in the classical sense. Also, its practical realization is complicated by the fact that we do not know the laws of change in the velocity $\mathbf{V}(\tau)$ of the solidification front and the normal $\mathbf{n}_\omega(\tau)$ to the external boundary of the skin. The direction of the boundary is impossible to determine even on a crust extracted from the melt, due to disturbances of the position of the external boundary by fluctuation errors associated with heat flow through the body of the blade and the skin after the cessation of blowing, nonuniformity of the temperature field in the melt, the presence of convective flows in the melt during extraction of the encrusted blade, etc.

It should be noted that the temperature of the blade wall is usually not measured because the placement of temperature sensors in the rather thin walls of the blade may intolerably distort the heat pattern. Also, there is a high probability that the sensors would be damaged when the crust is removed from the blade.

In principle, it is possible to determine $\mathbf{V}(\tau)$ by repetition of the experiment with the same air flow rate but different blowing times. However, it is difficult to reproduce cooling conditions in commercial trials.

Problem (1)-(8) can be simplified by taking the following into consideration.

A zone of columnar crystals is formed on the rapidly cooling surface during crystallization. The direction of growth of these crystals is opposite the direction of heat flow [6] and coincides with the direction of maximum thermal conductivity of the crystals, i.e., with the direction normal to the isothermal surface. This fact was also used in (8). The above means that the crystalline structure of the skin can be used to determine the direction of the normal $\mathbf{n}_\omega$ to the boundary ∂D_Σ^*.

Figure 2 shows a photograph of the macrostructure of a zinc skin formed on the outside surface of a full-size blade in the case where internal cooling was intensified on the concave side and significant thickening of the skin took place. The grain boundaries are evident in the photograph. The axes of the grains, which contain crystals with similar orientations, are themselves oriented normal to the fixed position of the solidification front.

We will assume that the grain boundaries are rectilinear, the thermophysical properties inside the grains are uniform, and the heat removed from the grains passes only through their bases – the sections of the surface intercepted by adjacent grain boundaries. Given these

Fig. 2. Macrostructure of the zinc crust.

conditions, we can examine heat transfer processes occurring in adjacent grains which are independent of one another. We can therefore reduce the two-dimensional IHP to a finite number of unidimensional problems in subregions of the region Σ.

Solution of the unidimensional IHP's requires a priori knowledge of the dependence of the velocity $V(\tau)$ of the solidification front on the time. This velocity can be found by determining the coordinate $\xi(\tau)$ of the front from the solution of the unidimensional Stefan problem:

$$\rho(r)C(r)\frac{\partial T(r,\tau)}{\partial\tau}=\frac{1}{r^M}\frac{\partial}{\partial r}\left(r^M\lambda(r)\frac{\partial T(r,\tau)}{\partial r}\right),\ R_1\leqslant r\leqslant\xi(\tau), \tag{14}$$

where M = 0 or 1, respectively, for plane and crylindrical walls:

$$T(r,0)=T_{so},\ R_1\leqslant r\leqslant R_2,\ \xi(0)=R_2, \tag{15}$$

$$\lambda(R_1)\frac{\partial T(r,\tau)}{\partial r}\bigg|_{r=R_1}=\alpha(T(R_1,\tau)-T_s), \tag{16}$$

$$\lambda(\xi)\frac{\partial T(r,\tau)}{\partial r}\bigg|_{r=\xi}=\rho Q\frac{\partial\xi}{\partial\tau}, \tag{17}$$

$$T(\xi,\tau)=T_{so}, \tag{18}$$

where $\rho(r)$, $C(r)$, $\lambda(r)$ are functions which are constant over time and continuous with respect to the coordinates. These functions take the values ρ_1, C_1, and λ_1 at $r \in [R_1, R_2 - \varepsilon]$, and ρ_2, C_2, and λ_2 at $r \geqq R_2 + \varepsilon$ and change linearly at $r \in (R_2 - \varepsilon, R_2 + \varepsilon)$. It was suggested in [7] that discontinuities in the thermophysical properties of materials be smoothed by monotonic relations, such as fourth-degree polynomials. Numerical experiments involving smoothing with linear and polynomial relations showed that the results of the calculations were reliable for practical purposes.

We introduce the variables

$$\left.\begin{aligned}&\Theta=\frac{T-T_s}{T_{so}-T_s},\ \delta=R_2-R_1,\ y_1=\frac{R_1}{\delta},\ y_2=\frac{R_2}{\delta},\ y_3=\frac{\xi}{\delta},\ z=\frac{r}{\delta},\\&t=\frac{\tau}{\tau_0},\ \mathrm{Bi}=\frac{\alpha\cdot\delta}{\lambda_1},\ \mathrm{Fo}=\frac{\lambda_2}{C_2\rho_2}\frac{\tau_0}{\delta^2},\ \mathrm{Ste}=\frac{C_2(T_{so}-T_s)}{Q},\end{aligned}\right\} \tag{19}$$

where τ_0 is a certain time scale.

In the new variables, Eqs. (14)-(18) take the form:

$$\frac{\delta^2\rho(z)C(z)}{\tau_0}\frac{\partial\Theta(z,t)}{\partial t}=\frac{1}{z^M}\frac{\partial}{\partial z}\left(z^M\lambda(z)\frac{\partial\Theta(z,t)}{\partial z}\right),\ y_1\leqslant z\leqslant y_3, \tag{20}$$

$$\Theta(z,0)=1,\ y_1\leqslant z\leqslant y_2, \tag{21}$$

$$\frac{\partial\Theta(z,t)}{\partial z}=\mathrm{Bi}\,\Theta(z,t),\ z=y_1, \tag{22}$$

$$\frac{\partial y_3(t)}{\partial t}=\mathrm{Ste}\cdot\mathrm{Fo}\frac{\partial\Theta(z,t)}{\partial z},\ z=y_3, \tag{23}$$

$$\Theta(y_3,t)=1. \tag{24}$$

We introduce a grid of nodes equidistant from z into the theoretical region. Let H be the mesh of the grid. We number the nodes beginning from the boundary of the region $z = y_1$

as 1, 2, ..., N, N + 1, ..., NN, NN + 1, where N + 1 is the number of the node coinciding with the boundary $z = y_2$ and NN + 1 is the number of the node coinciding with the solidification front.

We replace the derivatives in (20)-(24) by finite-difference relations

$$\frac{\partial\Theta}{\partial t}=\frac{\Theta_i^{j+1}-\Theta_i^j}{\Delta t^j},\quad \frac{\partial\Theta}{\partial z}=\frac{\Theta_{i+1}^{j+1}-\Theta_i^{j+1}}{H},$$

$$\frac{\partial}{\partial z}\left(z^M\lambda(z)\frac{\partial\Theta}{\partial z}\right)=\frac{z_{i+1/2}^M\lambda_{i+1/2}(\Theta_{i+1}^{j+1}-\Theta_i^{j+1})-z_{i-1/2}^M\lambda_{i-1/2}(\Theta_i^{j+1}-\Theta_{i-1}^{j+1})}{H^2}, \tag{25}$$

$$A_i\Theta_{i+1}^{j+1}+B_i\Theta_i^{j+1}+C_i\Theta_{i-1}^{j+1}+D_i=0,\quad i=2,\ 3,\ \ldots,\ NN;$$

where $\lambda_{i+1/2} = \lambda_{i+1} + \lambda_i/2$, $z_i = y_i + H(i - 1)$; i is the number of the node; j is the number of the time layer; Δt^j is the time step, which depends on j.

After insertion of (25) into (20)-(24) and the execution of some simple transformations, we obtain an implicit T-shaped finite-difference scheme with a three-diagonal matrix:

$$j=1,\ 2,\ 3,\ \ldots, \tag{26}$$

$$\Theta_i^j=1,\ j=1,\ 1\leqslant i\leqslant N+1, \tag{27}$$

$$\Theta_2^{j+1}=\Theta_1^{j+1}(1+H\,\mathrm{Bi}), \tag{28}$$

$$\Theta_{NN}^{j+1}=1-\frac{H^2}{\mathrm{SteFo}\Delta t^j}, \tag{29}$$

where the coefficient A_i and C_i depend on the form of the region Σ and B_i and D_i also depend on the time Δt^j.

The resulting nonlinear system of algebraic equations is solved by a method similar to that described in [7]. It is assumed that the solidification front moves over integral nodes of the grid, and a value of Δt^j which simultaneously satisfies (26) and (29) is determined by iteration.

With a known Δt^j, the system of difference equations is solved by the trial run method. The problem is solved first for the coefficients from the boundary $z = y_1$ to the boundary $z = y_3$ and then for the temperature Θ_i in the opposite direction.

When the temperatures Θ_{NN} at the nodes closest to the solidification front obtained from (26) and (29) do not conform to the specified accuracy, we refine the time Δt^j of movement of the front from one node to another and repeat the cycle.

A program for numerical solution was written in FORTRAN for the ES computer.

We performed a numerical study of the laws governing crystallization of zinc on a flat surface and on cylindrical walls made of a heat-resistant blade alloy with a wall thickness δ = 1-4 mm. The radius of curvature of the inside surface of the cylindrical wall R_1 = 2-32 mm. Solidification took place on both the concave and convex sides. The range of the heat-transfer coefficient α was 250-12,000 $W/m^2\cdot K$, which is somewhat broader than the range seen in blades undergoing cooling. The air temperature was 20°C.

We analyzed laws of the change, over time, of the actual q_a and nominal q_n heat fluxes. These fluxes were determined from the formulas:

$$q_a=\alpha(T(R_1,\ \tau)-T_s), \tag{30}$$

$$q_n=\rho QS(\tau)/(L\tau), \tag{31}$$

where $S(\tau)$ is the cross-sectional area of the crust at the moment of time τ; L is the perimeter of the convectively cooled surface of the wall.

Figure 3 shows the functions $q_a(\tau)$ and $q_n(\tau)$ for a wall 2 mm thick when there are significant differences in cooling conditions. For greater clarity, the heat fluxes are referred to the maximum value of convective heat flux $q_0 = \alpha(T_{so} - T_s)$ seen at the moment of time τ = +0.

The numerical studies established the following: 1) the actual heat flux monotonically decreases over time; 2) the nominal heat flux increases and reaches a maximum at Fo = 2-5; it then monotonically decreases, always remaining less than the actual heat flux; 3) the differences between q_a and q_n are no greater than 15%; 4) at Fo > 5, it can be assumed with an

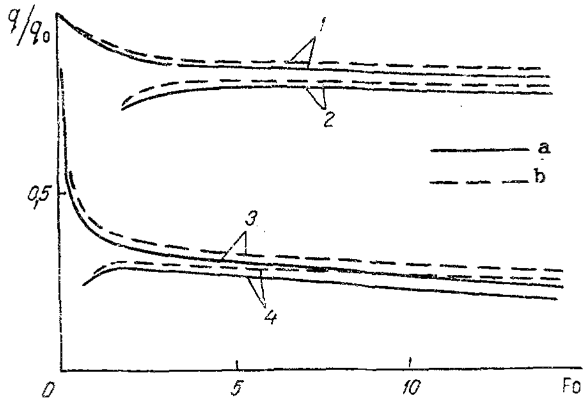

Fig. 3. Time change of the actual q_a and nominal q_n heat fluxes (a – flat wall, δ = 2 mm; b – cylindrical wall, R_1 = 30 mm, R_2 = 32 mm: 1 – q_a/q_0 with α = 1160 W/(m²·K); 2 – q_n/q_0 at α = 1160 W/(m²·K); 3 – q_a/q_0 at α = 11,600 W/(m²·K); 4 – q_n/q_0 at α = 11,600 W/(m²·K).

accuracy sufficient for practical purposes that $S(\tau)/\tau$ = const for the entire range of the parameters studied. The latter fact makes it possible to use the measured coordinate of the solidification front to calculate $\partial\xi/\partial\tau$ in the Stefan condition (8).

NOTATION

λ_D, λ_Σ, thermal conductivity in the regions D and Σ, respectively; a_D, a_Σ, diffusivity in the regions D and Σ, respectively; C, heat capacity; ρ, density; Q, heat of crystallization; q, heat flux; T, temperature; τ, time; x, y, r, coordinates; δ, wall thickness; R_1, R_2, coordinates of the convectively cooled and external surfaces of the wall, respectively; $\xi(\tau)$, coordinate of the solidification front; α, heat-transfer coefficient; $Fo = a_D\tau/\delta^2$, Fourier number; T_{so}, solidification temperature; $V(\tau)$, velocity of the solidification front; n_ω, normal to the contour.

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